NASA TECHNICAL
MEMORANDUM

NASA TM X-53454

April 27, 1966

N66 30118
(ACCESSION NUMBER)
39
(PAGES)
TMX-53454
(NASA CR OR TMX OR AD NUMBER)
(THRU)
1
(CODE)
33
(CATEGORY)
FACILITY FORM 602

NASA TM X-53454

A NEW CONCEPT TO THE GENERAL UNDERSTANDING OF THE EFFECTS OF LONGITUDINAL CONDUCTION FOR MULTISTREAM COUNTERFLOW HEAT EXCHANGERS

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GPO PRICE $ ____

CFSTI PRICE(S) $ ____

Hard copy (HC) # 2.00

Microfiche (MF) .50

ff 653 July 65

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ABSTRACT

A general understanding of the effect of longitudinal conduction on fluid temperatures for a multistream heat exchanger is obtained from a plot of a characteristic function. Mathematically, three corresponding sets of data are found to be real and distinct except for one possible pair of zero-double-roots for the balanced flow case. This assures four different forms of solutions. The function suggests the importance of the diagonal terms in the exact solutions. Therefore, an order of magnitude analysis is made on the basis of this function and the diagonal terms. A new descriptive parameter is derived and the importance of flowrate can be seen. An inefficiency plot was determined from an example analysis of a two-stream symmetric case. Performance equations, correction formula, and quantitative design charts are presented. The effect of large temperature differences on the ends is discussed mathematically.

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DEFINITION OF SYMBOLS

Symbol	Definition	Units
A	Heat transfer area	ft^2/ft of length
A_c	Total area subject to longitudinal conduction	ft^2
a_{nk}	Defined by equation (55) $1/(1 + \lambda_K/\alpha_n)$	
c	Specific heat of fluids	$Btu/lb\ {}^{-0}F$
C'_o, C_o, C_{wk}, C	- Integration constants	
$F(\lambda)$	Characteristic function defined by equation (4)	
$f(N_K)$	$[1/\ 2p(1-p)]$ defined by equation (64)	
$g(N_K)$	$[1/\ 2p(1+p)]$ defined by equation (65)	
h	Heat transfer coefficient	$Btu/hr\text{-}ft^2\text{-}{}^0F$
I_ϵ	$\xi_N - \xi_c$, Loss of effectiveness due to the effect of longitudinal conduction	
K	Conductivity of wall materials	$Btu/Hr\text{-}ft\text{-}{}^0F$
K_1, K_2	Conduction heat flux at warm (or cold) end	
L	Length of the exchanger	ft
m	Mass flow rate	lbm/hr
N_K	$(KA_c/mc)\alpha$, Modified longitudinal-conductivity-flowrate-ratio, dimensionless	
N_L	NTU $= \alpha(L/2)$, Modified length p parameter, dimensionless	
N_B	$KA_c/(mc)_m L$, defined by equation (46)	

Symbol	Definition	Units
N_{Mo}	$(L^2/2)(hA/KA_c)$	
$p\ (N_K)$	$(1 + 2/N_K)^{1/2}$	
Q	Total heat transferred	Btu/hr
$R\ (N_K)$	$(1\text{-}f + g)^{1/2}$	
$S\ (N_K)$	$(1\text{-}f + g)^{1/3}$	
T_n	Fluid temperatures, dependent variables	^0F
T_w	Wall temperature, dependent variable	^0F
T''_p	Inlet temperature at warm end	^0F
T'_q	Inlet temperature at cold end	^0F
U	Overall heat transfer coefficient	Btu/hr- F-ft^2
WOLC	The case where the effect of longitudinal conduction is neglected	
WLC	The case where the effect of longitudinal conduction is included	
X	Independent variable in length direction	ft
Y	Dependent variable in $F(\lambda)$ versus λ plot	

<u>Greek Letters</u>

α_n	$(hA/mc)_n$, Asymptotic ratios	
β_n	Roots for the case of WOLC	

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition	Units
λ_n	roots for the case of WLC	
ξ	$(T''_p - T'_p)(mc)_p/(T''_p - T'_q)(mc)_m$, Effectiveness on the thermodynamics base for two-stream exchanger	
θ	$(T - T''_p)/(T''_p - T'_q)$, Dimensionless temperature field	

Subscripts

C	The case of WLC	
infl	Point of inflection	
k	Subscripts related to number of roots	
m	Minimum value	
max	Maximum value	
n	Number of streams, or total number of streams	
N	The case of WOLC	
o', o	Number of two special roots	
p_i (+)	Number of streams flowing in positive direction	
q_j (-)	Number of streams flowing in negative direction	
r	Matrix rotor in regenerator	

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SUMMARY

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Considerable interest has recently been given to the design of gas flow heat exchangers with large heat transfer areas operating at low flow rates. However, these design conditions tend to magnify the effect of longitudinal conduction. The mathematical description of two-stream and multistream counterflow heat exchangers, including longitudinal conduction in the separating walls, by the authors, and work of others, have produced exact solutions or approximations that are too involved algebraically. Consequently, the relationship between the mathematical results and the physical phenomena could not be as clearly expressed as with conventional methods. A new concept is presented in which a plot of a characteristic function is used to obtain a general understanding of the effect of longitudinal conduction for both the two-stream and multistream cases.

The function suggests the importance of the diagonal terms in the exact solutions. Therefore, an order of magnitude analysis is made on the basis of this function and the diagonal terms. A new descriptive parameter was derived and the importance of flowrate can be seen. An inefficiency plot was determined from an example analysis of a two-stream symmetric case. Performance equations, correction formula, and quantitative design charts are presented. The effect of large temperature differences on the ends is discussed mathematically.

INTRODUCTION

Kays and London [1] pointed out that considerable interest has recently been given to the design of gas flow heat exchangers with large heat transfer areas operating at low flow rates. Of course, these design conditions tend to magnify the effect of longitudinal conduction; because of this, the authors have considered the mathematical description of two-stream (Ref. 2) and multistream (Ref. 3) counterflow heat exchangers including longitudinal conduction in the separating walls. However, these developments, as well as those of previous authors such as Dr. Hahnemann [4], and Landau and Hlinka [5], have produced exact solutions or approximate techniques that are too involved algebraically. Consequently, the relationship between the mathematical results and the physical phenomena could not be expressed as clearly as with the conventional methods. A new concept is presented in this report in which a plot of a characteristic function is utilized to obtain a general understanding of the effect of longitudinal conduction for both the two-stream and multistream cases. A simplified method based on this function is presented.

It is only necessary that $(mc)_r/(mc)_m > 5$ for the periodic flow to approach the direct transfer type behavior. The analysis and physical interpretations presented will aid the reader in the analysis and interpretation of periodic flow regenerators for which only approximate and numerical results are presently available (Ref. 6 and 7).

MATHEMATICAL MODEL

DIFFERENTIAL EQUATIONS THAT DESCRIBE THE SYSTEM

The differential equations that describe an n-stream heat exchanger were derived by Sze and Cimler [8]. They assumed infinite conductivity normal to flow for a reversing heat exchanger. Application of these equations to plate and fin type heat exchangers, including finite conductivity in the direction of flow, is discussed in Reference 3. The mathematical description of the two-stream

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exchanger is, of course, a special case of the general system. The differential equations for the multistream case, including longitudinal conduction in the separating walls, are written as follows:

$$(\underline{+}\, mc)_n \frac{dT_n}{dX} + (hA)_n (T_n - T_w) = 0 \tag{1}$$

$$\sum_{n=1}^{n} (\underline{+}mc)_n \frac{dT_n}{dX} = KA_c \frac{d^2T_w}{dX^2} \tag{2}$$

The boundary conditions for constant heat flux at the ends of the separating walls and constant fluid inlet temperatures are written as follows:

$$\begin{aligned} X &= 0 \quad T_{pi} = T_p'' \quad \frac{dT_{w(o)}}{dX} = K_1 \\ X &= L \quad T_{qj} = T_q' \quad \frac{dT_w(L)}{dX} = K_2 \end{aligned} \tag{3}$$

CHARACTERISTIC FUNCTION

The following form of the characteristic equation is found to be significant. It is derived in Appendix A.

$$F(\lambda) = \sum_{n=1}^{n} \frac{1}{\lambda/(hA)_n + 1/(\pm mc)_n} = KA_c\lambda \tag{4}$$

$$\lambda_o' = 0 \tag{5}$$

The roots of equation (4) are determined graphically. An example is given for a four-stream unbalanced flow heat exchanger. Figure 1 is the $F(\lambda)$ versus λ plot for this case. There are $(n + 1) = 5$ discontinuous curves. The intersections of the curves with the λ-axis are the roots for the WOLC case, and the intersections of the curves with $Y = KA_c\lambda$ are the roots for the WLC case. The total conductivity (KA_c) is the slope of the straight line that shifts the roots from $Y = 0$ to $Y = KA_c\lambda$.

The form of the solution of the set of differential equations depends on the type of roots that are obtained from equation (4).

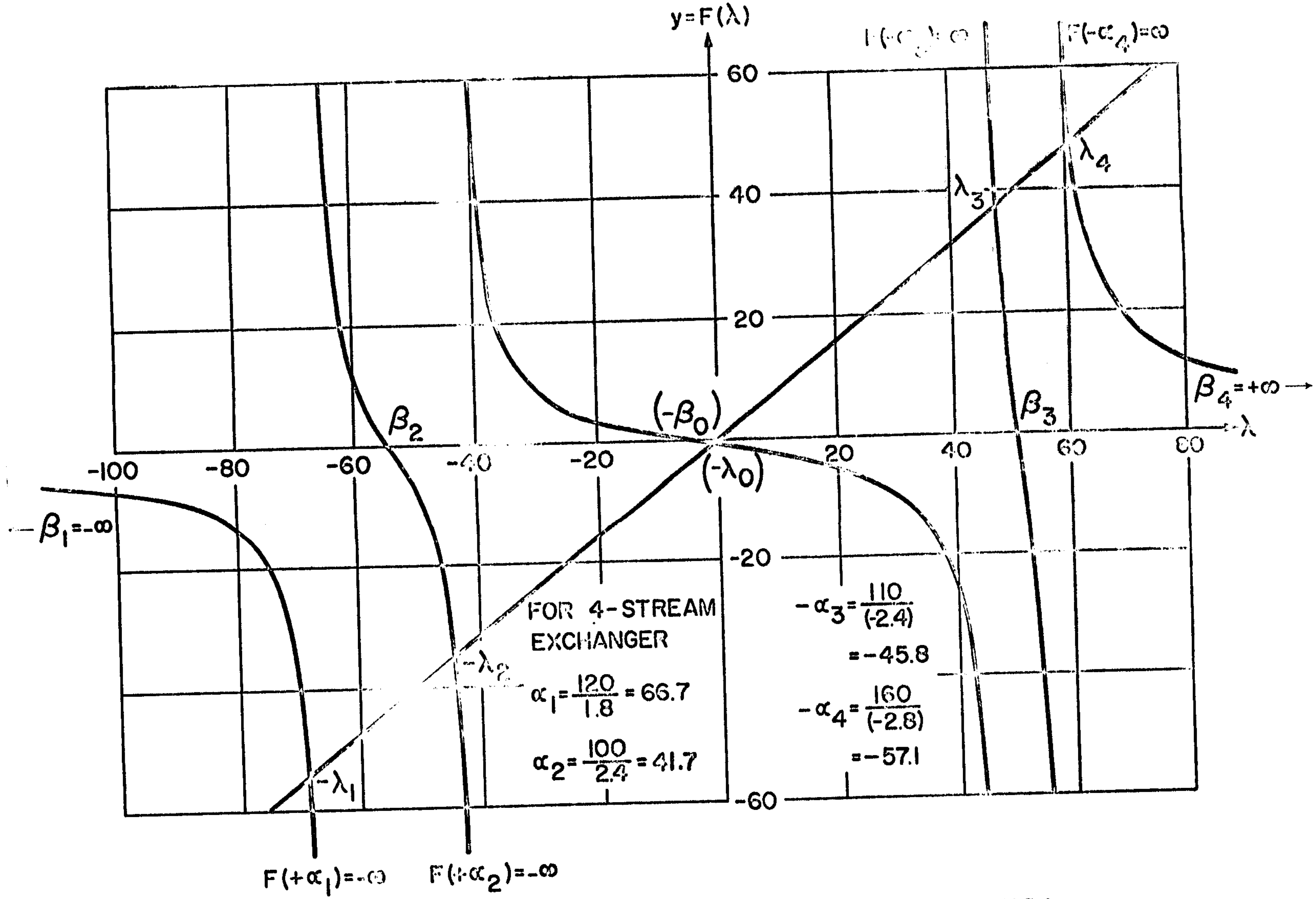

FIGURE 1 CHARACTERISTIC FUNCTION F(λ) VERSUS λ PLOT FOR FOUR-STREAM HEAT EXCHANGERS

1. If all the roots are real, the solution is written as follows:

$$T = \sum_{k=1}^{n} C_k \exp(\lambda_k X) \tag{6}$$

2. Since the coefficients of the differential equations are all real, complex roots appear as conjugate pairs. If only one pair of the roots are complex numbers, the form of the solution will be

$$T = (C_1 \cos \lambda_1 X + C_2 \sin \lambda_1 X) \exp(\lambda_2 X) + \sum_{k=3}^{n} C_k \exp(\lambda_2 X) \tag{7}$$

3. If all the roots are real, but one pair are double roots, that is

$$\lambda_1 = \lambda_2 = \lambda$$

the solution is

$$T = C_1 \exp(\lambda X) + C_2 X \exp(\lambda X) + \sum_{k=3}^{k} C_k \exp(\lambda_k X) \tag{8}$$

If three roots are equal, the solution is

$$T = C_1 \exp(\lambda X) + C_2 X \exp(\lambda X) + C_3 X^2 \exp(\lambda X) + \sum_{k=4}^{k} C_k \exp(\lambda_k X) \tag{9}$$

4. Any other combination of the following terms

$$\exp(\lambda X),\ X \exp(\lambda X),\ X^2 \exp(\lambda X),\ \ldots \cos \lambda X, \tag{10}$$

$\sin \lambda X$ ... may appear in the solutions.

Although other mathematical proof can be presented, it is easy to explain the possible types of roots from the plot of the characteristic function. If n values of $\alpha_n = (hA/mc)_n$ are real, there are (n+ 1) curves for which $-\infty \leqq F(\lambda) \leqq \infty$, $-\infty \leqq F(\lambda) \leqq 0$, $0 \leqq F(\lambda) \leqq +\infty$, and each curve intersects with $Y = 0$ and $Y = KA_c\lambda$ producing n+ 1 real roots. Therefore, there will be no complex roots, double roots, or triple roots, and no solutions in the form of equations (7), (9), and other combinations. But the form of equation (8) is still possible since the central curve may pass through the origin, i. e., one of the roots is zero. If $\lambda_o = 0$, then from equation (5), $\lambda'_o = 0$, $\lambda'_o = \lambda_o = 0$. The characteristic equation may, therefore, have one pair of double roots equal to zero, which produces a solution of the form of equation (8). The simplified form that results is

$$T = C_1 + C_2 X + \sum_{k=1}^{k} C_k \exp(\lambda_k X) \tag{10}$$

MATHEMATICAL SIGNIFICANCE OF BALANCED FLOW AND UNBALANCED FLOW

The term "unbalanced flow" for multistream heat exchangers is precisely defined as "the condition in which the sum of the time rates of heat capacity in the positive direction is not equal to that in the negative direction," that is

$$\sum_{n=1}^{i} (+\ mc)_n + \sum_{n=i+1}^{n} (-\ mc)_n \neq 0 \tag{11}$$

It follows for "balanced flow" that

$$\sum_{n=1}^{i} (+\ mc)_n + \sum_{n=i+1}^{n} (-\ mc)_n = 0 \tag{12}$$

The central curve of the $F(\lambda)$ versus λ plot passes through the origin for balanced flow and is slightly shifted for unbalanced flow. This is readily illustrated by substituting $\lambda = 0$ into the characteristic function to obtain

$$F\ (o) = \sum_{n=1}^{i} (+\ mc)_n + \sum_{i+1}^{n} (-\ mc)_n$$

Therefore,

$F\ (o) = 0$ for "balanced Flow"

$F\ (o) \neq 0$ for "unbalanced flow".

Since one pair of zero-double-roots occurs in balanced flow and all roots are distinct for unbalanced flow, the basic forms of the solutions for these two conditions are different. It is apparent [2] that two independent solutions exist.

TWO SPECIAL ROOTS

It can be concluded from this discussion that

1. for the case of balanced flow and WOLC:

   $\beta'_o = \beta_o = 0$

2. for the case of balanced flow and WLC:

   $\lambda'_o = \lambda_o = 0$

3. for the case of unbalanced flow and WOLC:

   $\beta'_o = 0$

   $\beta_o \neq 0$

   For the two-stream case:

$$UL \left(\frac{1}{(mc)_{II}} - \frac{1}{(mc)_{I}} \right) = \beta_o \tag{13}$$

4. for the case of unbalanced flow and WLC:

   $\lambda'_o = 0$

   $\lambda_o \neq 0$, $|\lambda_o|$ is smaller than $|\beta_o|$

THREE SETS OF ONE-ONE-ONE CORRESPONDENT DATA

If the two roots considered above are excluded, the other roots can be traced along each of the F (λ) versus λ curves (FIG 1). Three sets of data, α_n, β_n, λ_n, are in one-one-one correspondence with the following physical relations:

1. The asymptotic ratios: $\alpha_n = (hA/\underline{+}wc)_n$
$n = 1, 2, \ldots n$ (14)

If $(hAL)_n$ is used to replace $(hA)_n$, to make α_n dimensionless, α_n is just the conventional 2 $(NTU)_n$ values. From the $F(\lambda)$ versus λ plot it is found that $-\alpha_n$(s) are locations at $F(\lambda)$ becomes infinite.

2. The roots for WLC case: λ_n $n = 1, 2, \ldots n$. The λ_n(s) are the values of λ for which $F(\lambda)$ intersects the line $Y = KA_c\lambda$.

3. The roots for WOLC case: β_n $n = 1, 2, \ldots n$. The β_n(s) are the values of λ for which $F(\lambda)$ intersects the λ axis. It is interesting to note that for this case:

$\beta_1 = -\infty$ $\beta_n = +\infty$ (where n is the last stream)

If the flowrate and surface conductance of each stream are given, these three sets of data can be calculated from the characteristic function.

THE PHYSICAL IMPORTANCE OF EXTREME CONDITIONS

1. When the total conductivity in the direction of flow is infinite, longitudinal conduction dominates.

$$\lambda_n \rightarrow (-\alpha_n) \quad n = 1, 2, \ldots n$$

$$(1 + \lambda_n/\alpha_n) \rightarrow 0 \qquad (15)$$

$$\lambda_o \rightarrow 0 \text{ for unbalanced flow}$$

2. When the total conductivity in the direction of flow is finite, both conduction and convection are important.

3. When the conductivity in the direction of flow is zero, convection heat transfer dominates. For the WOLC case:

$$\lambda_n \rightarrow \beta_n \quad n = 1, 2, \ldots n$$

$$\lambda_1 \rightarrow (-\infty) = \beta_1 \qquad (16)$$

$$\lambda_n \rightarrow (+\infty) = \beta_n \text{ (n is the last stream in this term)}$$

$$\lambda_o \rightarrow \beta_o$$

GENERAL SOLUTIONS

All of the possible roots of the characteristic equation are now clear. The general solutions are readily obtained for the following cases:

1. for balanced flow and WLC:

$$T_w = C'_o + C_o X + \sum_{k=1}^{n} C_{wk} \exp(\lambda_k X) \tag{17}$$

$$T_n = C'_o + C_o (X - 1/\alpha_n) + \sum_{k=1}^{n} \frac{C_{wk}}{1 + \lambda_k/\alpha_n} \exp(\lambda_k X) \tag{18}$$

2. for unbalanced flow and WLC:

$$T_w = C'_o + C_o \exp(\lambda_o X) + \sum_{k=1}^{n} C_{wk} \exp(\lambda_k X) \tag{19}$$

$$T_n = C'_o + \frac{C_o}{1 + \lambda_o/\alpha_n} \exp(\lambda_o X) + \sum_{k=1}^{n} \frac{C_{wk}}{1 + \lambda_k/\alpha_n} \exp(\lambda_k X) \tag{20}$$

3. for balanced flow and WOLC:

$$T_w = C'_o + C_o X + \sum_{k=2}^{n-1} C_{wk} \exp(\beta_k X) \tag{21}$$

$$T_n = C'_o + C_o (X - 1/\alpha_n) + \sum_{k=2}^{n-1} \frac{C_{wk}}{1 + \beta_k/\alpha_n} \exp(\beta_k X) \tag{22}$$

4. for unbalanced flow and WOLC:

$$T_w = C'_o + C_o \exp(\beta_o X) + \sum_{k=2}^{n-1} C_{wk} \exp(\beta_k X) \tag{23}$$

$$T_n = C'_o + \frac{C_o}{1 + \beta_o/\alpha_n} \exp(\beta_o X) + \sum_{K=2}^{n-1} \frac{C_{wk} \exp(\beta_k X)}{1 + \beta_k/\alpha_n} \tag{24}$$

A general procedure for the determination of the (n+1) integration constants, C'_o, C_o, C_{wk}, from the boundary conditions is presented later.

A GENERAL UNDERSTANDING FROM A TWO-STREAM SYMMETRIC EXCHANGER

The temperature fields obtained from the exact solutions are necessary for the general understanding of the problem. However, the equations are too lengthy for design calculations. Therefore, the design techniques are usually based on dimensionless parameters that can be used to develop more convenient calculational procedures.

TWO DESIGN PARAMETERS FOR THE SYMMETRIC TWO-STREAM EXCHANGER ($h_1 = h_2$; $m_1 = m_2$)

A dimensionless parameter that includes the effect of longitudinal conduction is defined as the modified-longitudinal-conductivity-flowrate ratio, which is written as follows:

$$N_K = \alpha \frac{KA_c}{mc} = \left(\frac{KA_c}{mc}\right)\left(\frac{hA}{mc}\right) \tag{25}$$

The NTU number may be considered as the modified length, and written as follows:

$$N_L = NTU = \frac{UAL}{mc} = \left(\frac{hA}{mc}\right)\frac{L}{2} = \alpha \frac{L}{2} \tag{26}$$

The temperature fields, the effectiveness, and the temperature differences can be expressed by these two parameters. A formal derivation with the physical meaning for N_K will be presented later. However, their validity can be clearly seen from the expressions presented in the following paragraphs.

APPLICATION OF THE CHARACTERISTIC FUNCTION

The characteristic function for the two-stream symmetric case is:

$$F(\lambda) = \frac{1}{\frac{\lambda}{hA} + \frac{1}{mc}} + \frac{1}{\frac{\lambda}{hA} + \frac{1}{mc}} = KA_c\lambda \frac{1}{\lambda/\alpha + 1}$$

$$+ \frac{1}{\lambda/\alpha - 1} = N_K (\lambda/\alpha) \tag{27}$$

1. For the WOLC case

$$\frac{1}{\beta/\alpha + 1} + \frac{1}{\beta/\alpha - 1} = 0$$

According to equations (21) and (22)

$$T_w = C'_o + C_o X$$

$$T_n = C'_o + C_o (X \pm 1/\alpha) \quad n = 1, 2 \tag{28}$$

After evaluating the integration constants, the same results as Jakob [9] , are obtained.

2. For the WLC case, equation (27) is used to obtain

$$2 \lambda/\alpha = N_K (\lambda/\alpha) (\lambda^2/\alpha^2 - 1)$$

$$\lambda L = \pm 2 N_L (1 + 2/N_K)^{1/2} \tag{29}$$

Using equation (17) and (18), the temperature fields are found to be similar to those of equations (32), (33) and (34) given in reference 2 with the exception that they are in terms of N_L and N_K.

MATHEMATICAL EXPLANATION FOR THE INCREASE OF END TEMPERATURE DIFFERENCE FOR THE WLC CASE

If the mathematical expressions for the mean temperature difference and end temperature difference are considered, an interesting observation can be made. The derivation of these expressions is given in Appendix B.

$$\Delta\theta_{mean} = \frac{1 + N_K \tanh(pN_L)/(2pN_L)}{N_L + N_K \tanh(pN_L)/(2p) + N_K/2 + 1} \tag{30}$$

$\Delta\theta_{mean}$ is defined by the total heat transferred; that is,

$$Q = UAL\ \Delta\theta_{mean} \tag{31}$$

If it is compared with the end temperature difference $\Delta\theta(0)$ or $\Delta\theta(L)$, only one term is found to be different:

$$\Delta\theta(0) = \Delta\theta(L) = \frac{1 + N_K/2}{N_L + N_K \tanh(pN_L)/(2p) + N_K/2 + 1} \tag{32}$$

The temperature field plots in References 2 and 3 suggest that longitudinal conduction increases the end temperature difference. At low flowrate and high conductivity, a sudden change of fluid temperature appears at the exchanger ends. This condition can be explained with equations (30) and (32); $\Delta\theta_{mean}$ is always smaller than $\Delta\theta(0)$, because the value of $\tanh(pN_L)$ is smaller than that of (pN_L), and

$$\tanh(pN_L)/(pN_L) < 1$$

Therefore, the effect of longitudinal conduction completely disappears only when

$$\tanh(pN_L) = (pN_L) \tag{33}$$

$$\text{for } N_L\ (1 + 2/N_K)^{1/2} = 0$$

$$\text{i.e., } \frac{hA}{mc} \cdot \frac{L}{2} = 0$$

Physically, this implies that the total heat transfer surface (or the length) is zero, or the flowrate is infinite.

MATHEMATICAL EXPRESSION OF EFFECTIVENESS AND THE RELATION BETWEEN THE WLC AND WOLC CASES

The mathematical expression for the effectiveness of the symmetric two-stream case:

$$\xi_C = \frac{N_L + N_K \tanh (pN_L)/(2p)}{N_L + N_K \tanh (pN_L)/(2p) + N_K/2 + 1} \tag{34}$$

If equation (33) were true for all values of (pN_L) and it is substituted into equation (34), the following equation is obtained:

$$\xi_N = \frac{N_L + N_K N_L/2}{N_L + N_K N_L/2 + N_K/2 + 1} = \frac{N_L}{N_L + 1} = \frac{NTU}{NTU + 1} \tag{35}$$

Equation (34) is reduced to the conventional form (equation (1)).

BOUNDARY LINE BETWEEN THE WLC AND WOLC CASES

The derivation given above does not mean that the conventional approach would not be true if $\tanh (pN_L) \neq (pN_L)$. This can be explained by use of an example from the boundary layer theory. The velocity in the boundary layer approaches the stream velocity asymptotically. However, the boundary layer is thin and is defined as the distance from the wall where the velocity differs from the free stream velocity by 1 per cent. If this criterion is adopted, a boundary line between the WLC and WOLC cases can be found. An algebraic expression for this line for the symmetric case is

$$N_L = 30\, N_K(1/p + 1) - 1 \tag{36}$$

This equation approximately describes the values of N_L and N_K for which the effectiveness calculated from equation (34) will differ from that calculated from equation (35) by less than 1 per cent. It prescribes the region for which the designer may ignore the effect of longitudinal conduction [10].

The ξ-NTU chart with N_K as a parameter is illustrated in Figure 2. The curve for $N_K = 0$ is taken from Reference 1, or calculated from equation (35). These revised curves are useful for gas flow heat exchangers with large heat transfer areas operating at low flow rates.

The ξ-N_L chart with N_K as a parameter and the ξ-N_K chart with N_L as a parameter are shown in more detail in Figure 3 and Figure 4.

BOUNDARIES OF THE REGION OF INEFFICIENCY

Division of equation (30) by equation (32) gives the ratio $\Delta\theta_{mean}/\Delta\theta(0)$

$$\frac{\Delta\theta_{mean}}{\Delta\theta(0)} = \frac{N_K \tanh(pN_L)/(2pN_L) + 1}{N_K/2 + 1} \tag{37}$$

The designer will prefer to keep this ratio as large as possible. Unfortunately, the increase of $\tanh(pN_L)$ with respect to (pN_L) approaches 1 asymptotically for $(pN_L) > 2.7$, that is

$$(pN_L) > 2.7 \qquad 0.99 < \tanh(pN_L) < 1$$

and then

$$\frac{\Delta\theta_{mean}}{\Delta\theta(0)} \cong \frac{N_K/2pN_L + 1}{N_K/2 + 1} \tag{38}$$

Therefore, the deterioration of thermal performance becomes worse for

$$N_L\,(1 + 2/N_K)^{1/2} > 2.7 \tag{39}$$

or

$$\lambda L > 5.4$$

The dotted line in Figure 3 and Figure 4 of the ξ-N_L and ξ-N_K charts indicates conditions

$$N_L\,(1 + 2/N_K)^{1/2} < 2.7 \tag{40}$$

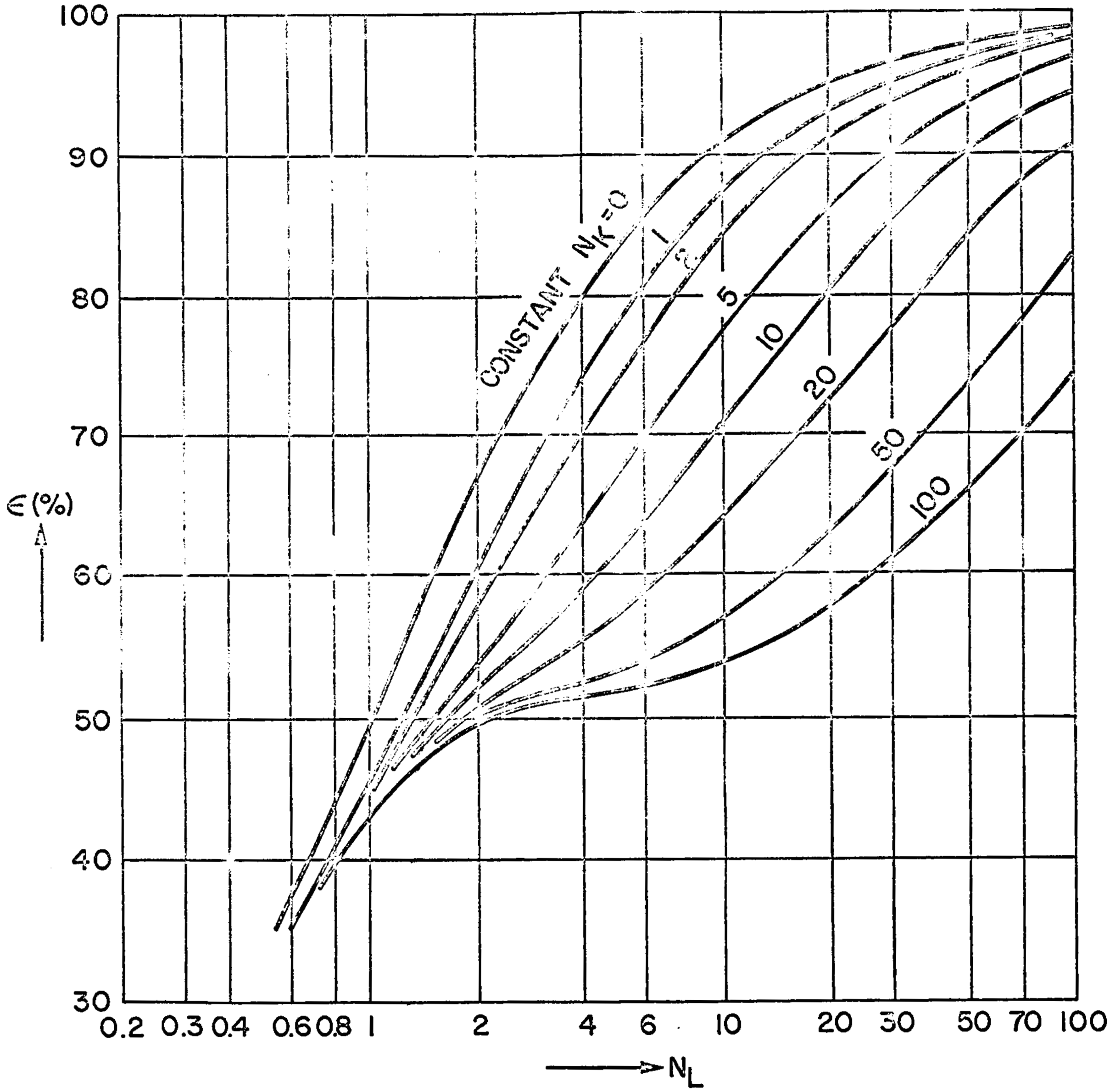

FIGURE 2 THE ξ-N_L CHART FOR TWO-STREAM SYMMETRIC HEAT EXCHANGERS

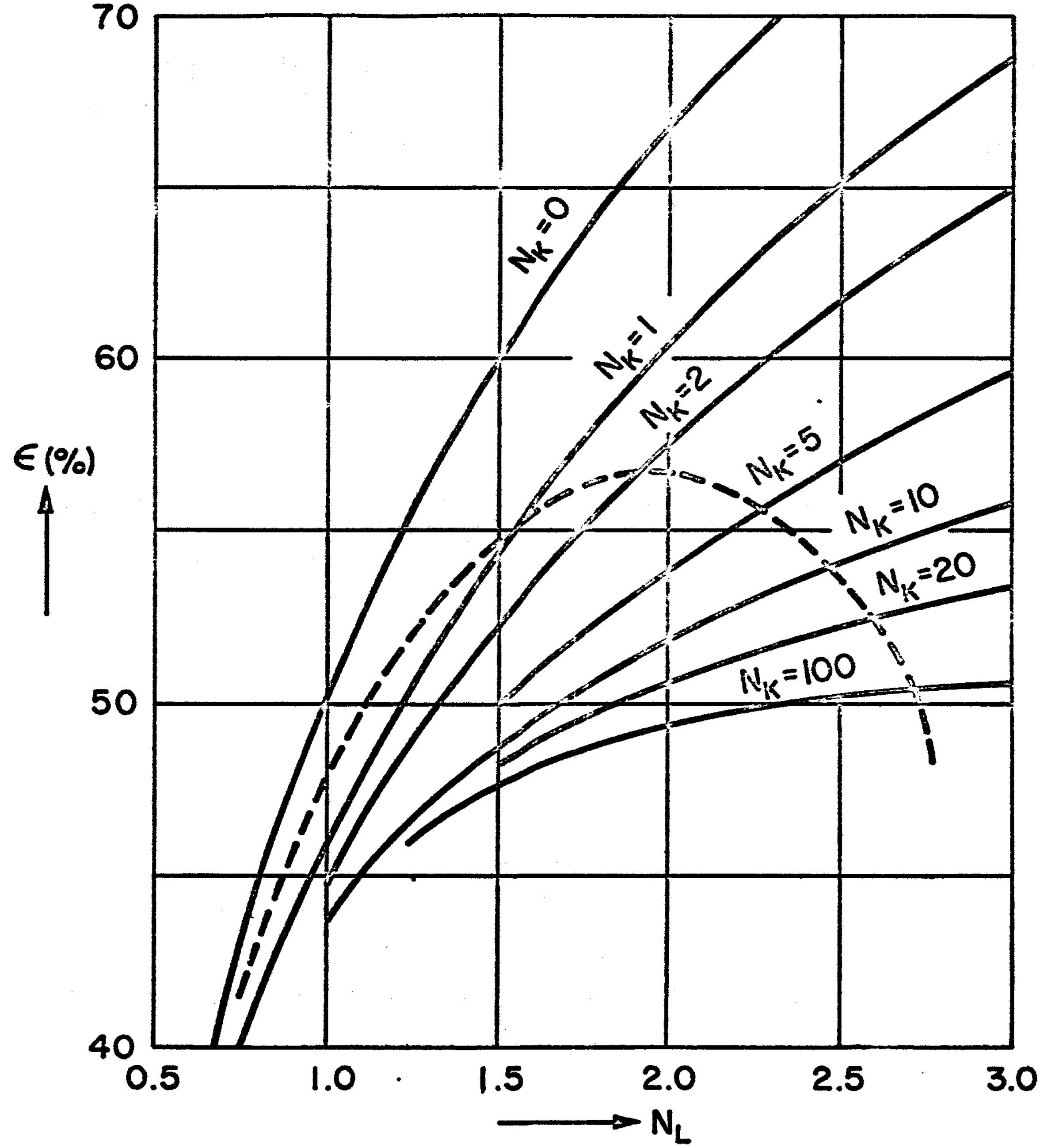

FIGURE 3 THE ξ-N_L CHART FOR TWO-STREAM SYMMETRIC HEAT EXCHANGERS

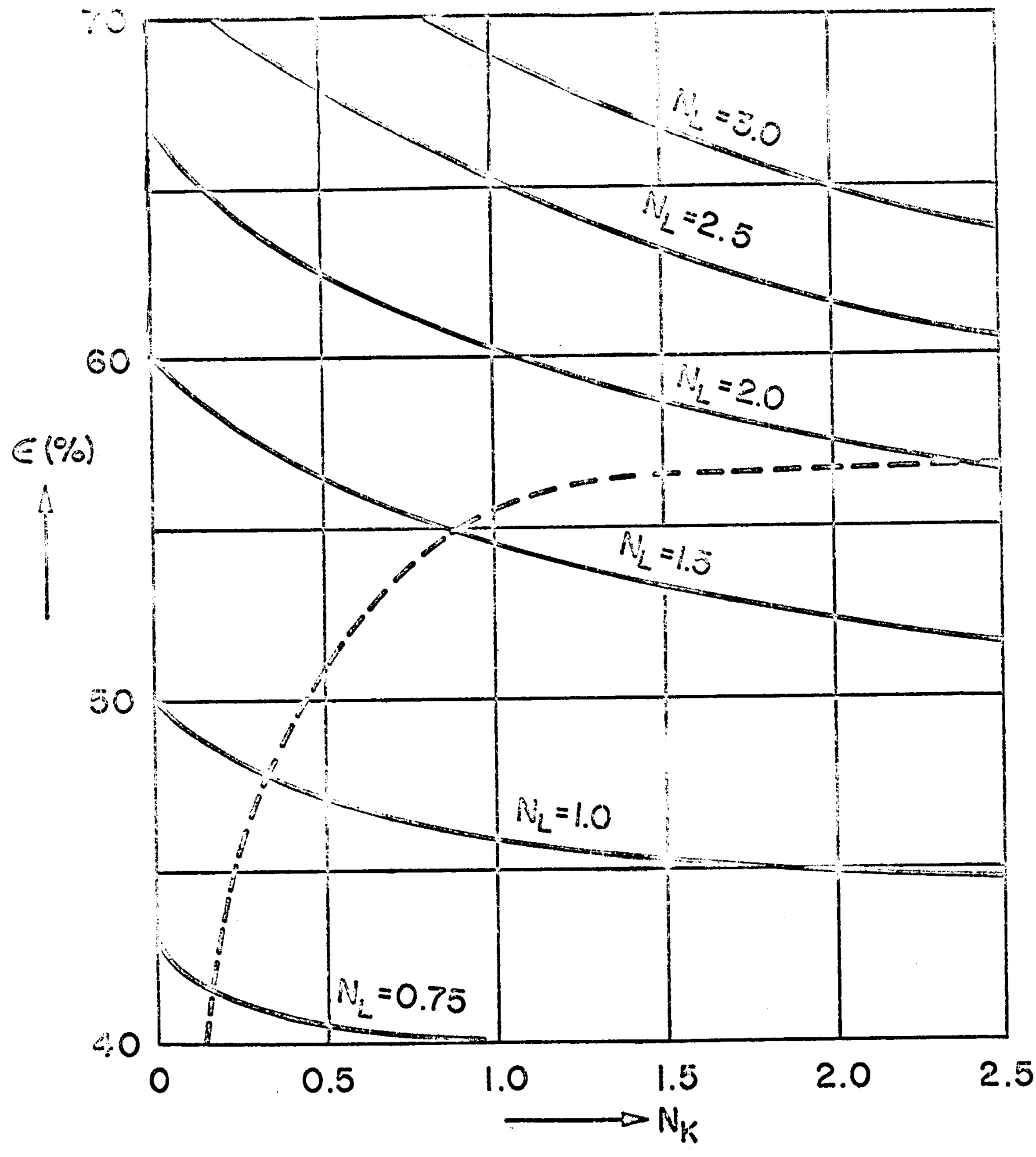

FIGURE 4 THE ξ-N_K CHART FOR TWO-STREAM SYMMETRIC HEAT EXCHANGERS

It is clear that in this region a large increase of N_K will not cause a large decrease of the effectiveness.

One of the boundaries of the region of inefficiency is defined as follows:

$$N_L \, (1 + 2/N_K)^{1/2} \triangleq 2.7 \tag{41}$$

In this case an abrupt drop of efficiency may occur for high N_K numbers.

SIMPLIFIED PERFORMANCE EQUATIONS

The region specified by equation (39) is not desirable. However, it inevitably occurs in practical problems. Fortunately, the mathematical expressions can be simplified, and the error introduced is very small:

$$\Delta\theta_{mean} = \frac{N_K/(2pN_L) + 1}{N_K/(2p) + N_K/2 + N_L + 1} \tag{42}$$

$$\Delta\theta(0) = \frac{N_K/2 + 1}{N_K/(2p) + N_K/2 + N_L + 1} \tag{43}$$

$$\xi = \frac{N_K/(2p) + N_L}{N_K/(2p) + N_K/2 + N_L + 1} \tag{44}$$

Comparison of numerical values calculated from the exact solution of equation (34) and simplified equation (44) is as follows:

N_K	N_L	ξ exact	ξ simplified
1.0	2.7	.666	.666
1.0	3.0	.687	.687
1.0	4.0	.741	.741
1.0	5.0	.779	.779
10.0	2.7	.547	.548
10.0	3.0	.557	.558
10.0	4.0	.588	.588
10.0	5.0	.6145	.6145

Further simplification can be obtained for large conduction effects:

1. If $N_K > 200$

$$\xi = \frac{N_K/2 + N_L}{N_K + N_L} = \frac{1 + N_B}{1 + 2N_B} \tag{45}$$

Where $N_B = (KA_c)/(mc)_m L \quad N_K = 2\, N_L\, N_B$ (46)

2. If $N_K \gg N_L$ and $N_K \gg 1$

$$\begin{aligned} \Delta\theta_{mean} &= 1/2N_L \\ \Delta\theta(0)' &= 1/2 \\ \xi &= 1/2 \end{aligned} \tag{47}$$

ORDER OF MAGNITUDE ANALYSIS

It has been shown with the use of equation (39), that $\lambda L = 5.4$ is a special value for the two-stream symmetric case. It is easy to generalize this method to unsymmetric, unbalanced flow and multi-stream cases.

First, it can be predicted from the F (λ) versus λ curve that

$$|\beta_n| > |\lambda_n| > |\alpha_n|$$

for each of these corresponding values. If α_m is the smallest of these asymptotic ratios, and

$$\alpha_m L > 5.4 \qquad \text{or} \qquad (N_L)_m > 2.7 \tag{48}$$

an order of magnitude analysis will lead to simpler expressions. It is realized that for the exponential function,

$$\exp(-\lambda_m L) > 200 \qquad \exp(+\lambda_m L) < 0.005$$

The algebraic equations that result will facilitate analytical work for large and more complicated heat exchangers. Numerical results are also sufficiently accurate if equation (48) is satisfied.

FOUR-STREAM EXAMPLE

A four-stream balanced-flow exchanger is considered as an example. The two special roots and the one-one-one correspondent data for the WLC and WOLC cases are:

Asymptotic Ratios (α_n)	Roots (λ_n)	Roots (β_n)
	$\lambda'_o = 0$	$\beta'_o = 0$
α_1	$-\lambda_1$	$-\beta_1 = \infty$
α_2	$-\lambda_2$	$-\beta_2$
$-\alpha_3$	λ_3	β_3
$-\alpha_4$	λ_4	β_4
	$\lambda_o = 0$	$\beta_o = 0$

Applying the boundary conditions represented by equation (3) to equations (17) and (18) for the WLC case, and if equation (48) is satisfied, the (n + 1) simultaneous equation for the six constants: C'_o, C_o, C_1-C_4 are simplified as follows:

$$\begin{bmatrix} 0 & \lambda_1 & \lambda_2 & 0 & 0 & +1 \\ 1 & a_{11} & a_{12} & 0 & 0 & -1/\alpha_1 \\ 1 & a_{21} & a_{22} & 0 & 0 & -1/\alpha_2 \\ 1 & 0 & 0 & a_{33}\, e^{\lambda_3 L} & a_{34}\, e^{\lambda_4 L} & L-1/\alpha_3 \\ 1 & 0 & 0 & a_{43}\, e^{\lambda_3 L} & a_{44}\, e^{\lambda_4 L} & L-1/\alpha_4 \\ 0 & 0 & 0 & \lambda_3\, e^{\lambda_3 L} & \lambda_4\, e^{\lambda_4 L} & 1 \end{bmatrix} \begin{bmatrix} C'_o \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_o \end{bmatrix} = \begin{bmatrix} K_1 \\ T''_p \\ T''_q \\ T'_q \\ T'_q \\ K_2 \end{bmatrix}$$

Equations 49, 50, 51, 52, 53, 54 - above.

Where $a_{nk} = 1/(1 + \lambda_k/\alpha_n)$ (55)

Solving C_o from equations (49), (50) and (51) in terms of C'_o and simultaneously from equations (52), (53) and (54), one readily obtains C'_o, C_o, C_1, C_2, C_3, C_4.

THE IMPORTANCE OF THE ROLE OF THE DIAGONAL TERMS - a_{kk}

It is concluded from the $F(\lambda)$ versus λ plot and equation (15), that

$$KA_c \to \infty, \quad (1 + \lambda_k/\alpha_k) \to 0 \tag{15}$$

$$a_{kk} = 1/(1 + \lambda_k/\alpha_k) \to \infty \tag{56}$$

which are the diagonal coefficients in equations (50) through (53). This set of coefficients will increase without limit, and when considering the characteristic function,

$$\frac{(mc)_1/(mc)_k}{1 + \lambda_k/\alpha_1} + \frac{(mc)_2/(mc)_k}{1 + \lambda_k/\alpha_2} + \cdots \frac{1}{1 + \lambda_k/\alpha_K}$$

$$+ \cdots = \alpha_k \left[\frac{KA_c}{(mc)_k}\right]\left[\left(\frac{\lambda}{\alpha}\right)_k\right]$$

It is found that when $KA_c \to \infty$, only one term, a_{kk}, in the left hand member of the equation remains significant. The other terms in the left hand member can be neglected when $a_{kk} > 100$. Equations (49) through (54) are further simplified as follows:

$$\begin{bmatrix} 0 & \lambda_1 & \lambda_2 & 0 & 0 & 1 \\ 1 & a_{11} & 0 & 0 & 0 & -1/\alpha_1 \\ 1 & 0 & a_{22} & 0 & 0 & -1/\alpha_2 \\ 1 & 0 & 0 & a_{33}\, e^{\lambda_3 L} & 0 & (L-1/\alpha_3) \\ 1 & 0 & 0 & 0 & a_{44}\, e^{\lambda_4 L} & (L-1/\alpha_4) \\ 0 & 0 & 0 & \lambda_3\, e^{\lambda_3 L} & \lambda_4\, e^{\lambda_4 L} & 1 \end{bmatrix} \begin{bmatrix} C'_o \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_o \end{bmatrix} = \begin{bmatrix} K_1 \\ T''_p \\ T''_p \\ T'_q \\ T'_q \\ K_2 \end{bmatrix}$$

Equations (57), (58), (59), (60), (61), and (62) - above.

Numerical calculations show that the diagonal terms are the predominant contributors to the abrupt change of the fluid temperatures at the inlets.

CORRECTION FORMULA FOR THE TWO-STREAM SYMMETRIC CASE

If the method used to obtain equations (49) through (54) is used for the two-stream symmetric case, the performance equations (42) through (44) will be obtained. This approach yields a somewhat different physical interpretation that may further simplify the problem. Using

$$\alpha_1 = \alpha, \ \alpha_2 = \alpha_4 = 0, \ \alpha_3 = -\alpha, \ \lambda_1 = -\lambda, \ \lambda_3 = \lambda$$

one obtains

$$\xi = \frac{N_L - f(N_K) - g(N_K)}{N_L - 2\, f(N_K) + 1} \tag{63}$$

where

$$f = \frac{1}{(2\lambda/\alpha)\,(1 - \lambda/\alpha)} = 1/(2p)\,(1-p) \tag{64}$$

$$g = 1/(2p)\,(1 + p) \tag{65}$$

It is noted that f is derived from $a_{kk} = 1/(1 + \lambda_k/\alpha_k)$, but g is derived from $a_{nk} = 1/(1 + \lambda_k/\alpha_n)$ for $n \neq k$. Therefore, f is usually more important, and g can be neglected for $N_K > 50$.

A correction formula can be based on the conventional approach of increasing the exchanger length to improve the effectiveness. If, for constant N_K, the loss of effectiveness due to longitudinal conduction could be compensated by an increase of N_L, the same effectiveness could be obtained for different lengths. Utilizing equations (35) and (63), the following correction formula can be determined.

$$N_{L,C} = f + g + N_{L,N}\,(1 - f + g) \tag{66}$$

for

$$N_{L,N}\,(1 + 2/N_K)^{1/2} > 2.7$$

If $N_K > 50$, and $N_{L,N} (1 + 2/N_K)^{1/2} > 2.7$, equation (66) can be further simplified to

$$N_{L,C} = f + N_{L,N} \ (1-f) \tag{67}$$

AN EXAMPLE CALCULATION USING THE CORRECTION FORMULA

1. It is of interest to consider the per cent increase of the length of a symmetric exchanger that is necessary to maintain the same effectiveness calculated from the equation for a WOLC case

$(mc) = 8$ Btu/hr^{0}F, $hA = 160$ Btu/hr ft ^{0}F,

$KA_c = 1.0$(Btu/hr^{0}F) . ft $L = 1$ ft

$N_{L,N} = (hA/mc) (L/2) = 10 > 2.7$ $KA_c = 1.0$,

$N_K = (hA/mc) (KA_c \ mc) = 2.5$

$p = (1 + 2/N_K)^{1/2} = 1.34164$ $f = 1/(1 - p) (2p) = -1.091$

$g = 1/2p(1 + p) = 0.159$ $N_{L,C} = f + g + N_{L,N} (1 - f + g) = 21.67$

$\Delta L\% = 21.67/10 - 1 = 116.7\%$

It appears to be impracticable to double the heat exchanger length in order to compensate for the loss due to effect of longitudinal conduction.

2. If the flowrate is doubled in this example, that is,

$(mc) = 16.0$ then $N_{L,N} = 5.0 > 2.7$

$N_K = 0.625$ $p = 2.04939$ $f = -0.232$ $g = 0.08$ $N_{L,C} = 6.408$

$\Delta L\% = 28.16\%$

Compensation by increasing the length appears to be more reasonable.

3. If the flowrate is reduced to one-half, that is (mc) = 4

$N_{L,N} = 20 > 2.7 \quad N_K = 10 \quad p = 1.09545 \quad f = -4.7819$

$g = 0.218 \quad N_{L,C} = 116.434 \quad \Delta L\% = 482\%$

Compensation by increasing the length becomes more reasonable. Therefore, the length correction method is practical only for the higher flowrate range. The 10 per cent correction for this effect as mentioned in Reference 8 would be reasonable only for some flowrate higher than 22.

THE IMPORTANT ROLE OF FLOWRATE AND THE DERIVATION OF THE CONDUCTIVITY-FLOWRATE-RATIO N_K

The above calculations illustrate that the effect of longitudinal conduction is very sensitive to flowrate. There is a mathematical derivation that supports this point. Recalling the diagonal terms, a_{kk}, which are over-weighted due to the effect of longitudinal conduction,

$$a_{kk} = 1/(1 + \lambda_k/\alpha_k)$$

For the two-stream symmetric case

$$\frac{1}{a_{kk}} = 1 + \lambda_k/\alpha_k = 1 - (1 + 2/N_K)^{1/2}$$

By use of the Binomial Theorem,

$$\frac{1}{a_{kk}} = 1 - \left[1 + 1/2\,\frac{2}{N_K} + \frac{1/2(-1/2)}{2}\left(\frac{2}{N_K}\right)^2 + \ldots\ldots\right]$$

If $N_K > 2$

$$\frac{1}{a_{kk}} \cong 1 - (1 + 1/N_K) = -\frac{1}{N_K}$$

Therefore,

$$a_{kk} = -N_K = - \frac{(hA)\,(KA_c)}{(mc)^2} = - \alpha\left(\frac{KA_c}{mc}\right) \qquad (68)$$

Equation (68) illustrates the conclusions that have already been observed in this and previous papers.

1. Flowrate has more effect than any other factor

2. The new parameter N_K, which stands for the modified conductivity-flowrate-ratio, will give a much better grasp of the effect of longitudinal conduction.

3. The parameter N_K is proportional to the total conductivity and the heat transfer coefficient and inversely proportional to the square of the flowrate.

LOWER BOUNDARY OF THE INEFFICIENCY REGION

It has been pointed out that the efficiency can be increased by increasing N_L. But, on the contrary, the discussion with equations (39), (40), and (41) indicates that if N_L is decreased below its critical value, the effect of longitudinal conduction will be decreased. Using equations (35) and (36), the loss of the effectiveness due to the effect of longitudinal conduction, I_ϵ, is found.

$$I_\epsilon = \xi_N - \xi_C = \frac{N_L}{1+N_L} - \frac{N_L - f - g}{1+N_L - 2f} = \frac{1 - f + g}{1+N_L - 2f} - \frac{1}{1+N_L} \qquad (69)$$

I_ϵ is plotted with respect to N_L, with N_K as a parameter (FIG 5 and 6). The loss of effectiveness is serious only in the middle range of N_L for constant N_K. Each N_K curve declines at both ends of N_L and reaches a maximum at a definite value of N_L.

LINES OF MINIMUM EFFICIENCY AND THE UPPER BOUNDARY OF THE INEFFICIENCY REGION

Since f and g are functions of N_K only, differentiation of I_ϵ with respect to N_L, yields the line of minimum efficiency.

$$\frac{dI_\epsilon}{dN_L} = \frac{-(1 + g - f)}{(1 + N_L - 2f)^2} + \frac{1}{(1 + N_L)^2} = 0$$

$$N_{L,\ \text{maximum}} = (1 - 2f - R)/(R - 1) \qquad (70)$$

Differentiating again

$$\frac{d^2 I_\epsilon}{dN_L{}^2} = 0, \qquad S(N_K) = (1 - f + g)^{1/3}$$

The point of inflection is obtained:

$$N_{L,\,infl} = (1 - 2f - S)/(S - 1) \tag{71}$$

Of course, at points of inflection, the curves change their directions of curvature. The rate of increase of the effectiveness due to the increase of N_L will gradually decline beyond these points. This line is considered as the upper boundary of the inefficiency region. At the same time, equation (41) may be used to define the lower region. The boundaries are indicated in Figure 5 and Figure 6. Figure 7 is a top view of the region which starts at $N_K = 0.1$.

A region affected by longitudinal conduction is also indicated in Figure 7. Note that equation (36) represents approximately the upper limit of the region at which the effectiveness differs within one per cent from the ideal WOLC case. As $N_L(1 + 2/N_K)^{1/2}$ becomes smaller,

$$\tanh\left[N_L\,(1 + 2/N_K)^{1/2}\right] \rightarrow N_L(1 + 2/N_K)^{1/2}$$

then the effect of longitudinal conduction becomes small and can be neglected for

$$N_L < 0.4 \tag{72}$$

This value of N_L can be used as the lower limit of the region for larger values of N_K.

A COMPARISON OF DATA WITH PREVIOUS WORK

It should be emphasized again that the inefficiency region is defined only on the basis of the effect of longitudinal conduction. It does not apply to the efficiency of the whole system. As shown in Figures 2 through 12 of Reference 1, the effectiveness decreases as N_L decreases in the WOLC case. If the effectiveness of the system

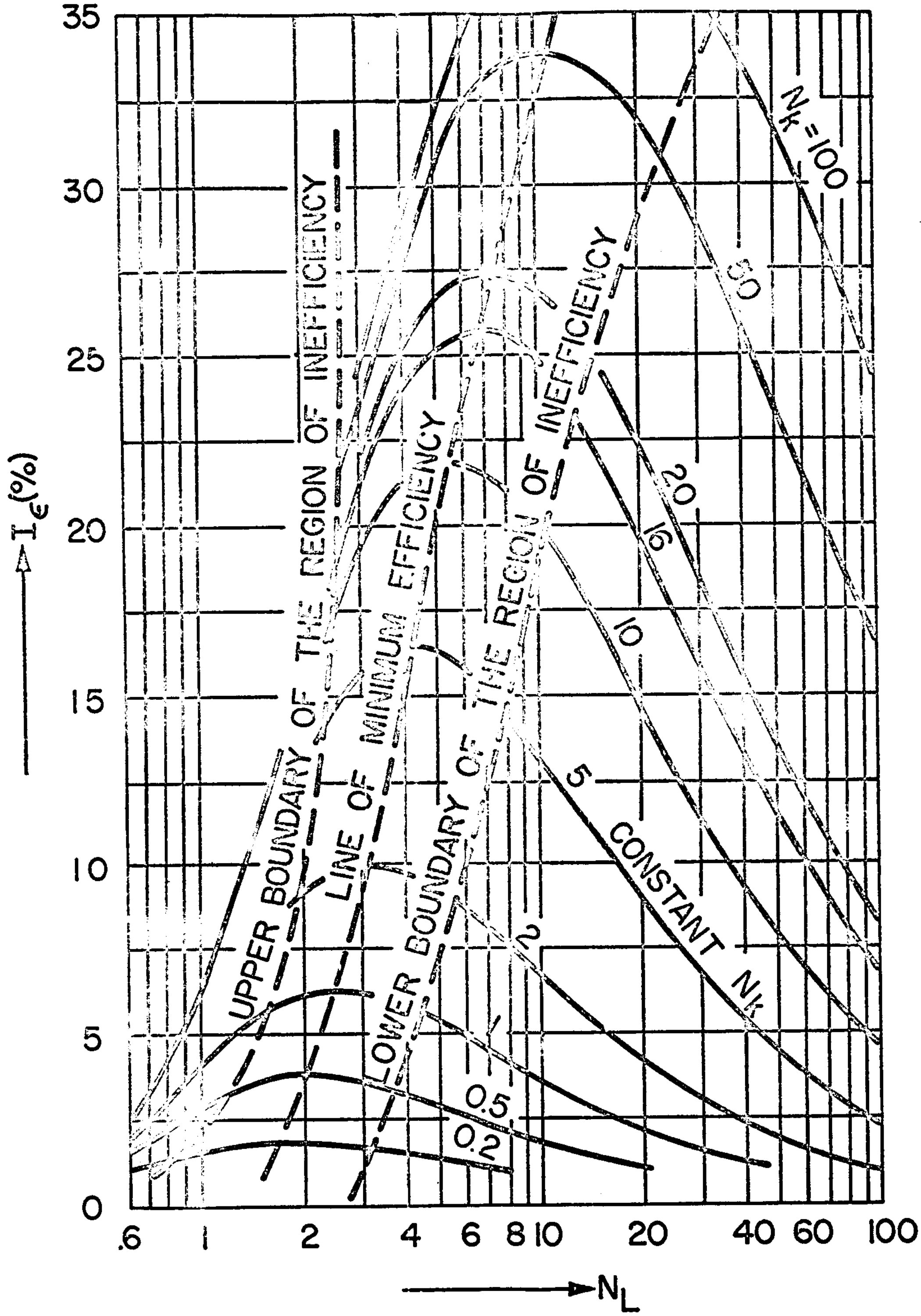

FIGURE 5 THE I_ϵ-N_L CURVES, THE INEFFICIENCY REGIONS FOR TWO-STREAM SYMMETRIC HEAT EXCHANGERS

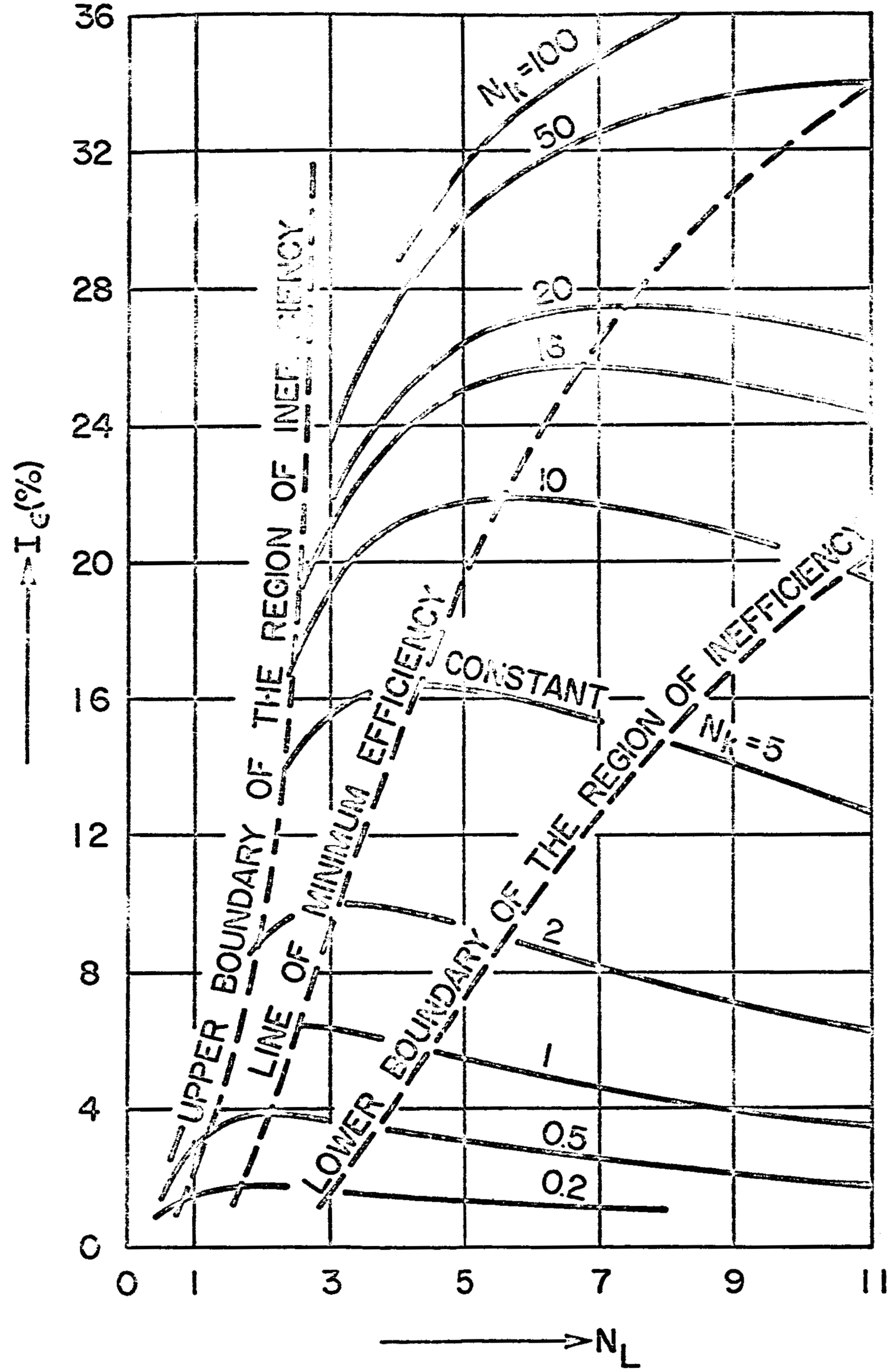

FIGURE 6 THE I_ϵ-N_L CURVES, THE INEFFICIENCY REGIONS FOR TWO-STREAM SYMMETRIC HEAT EXCHANGERS

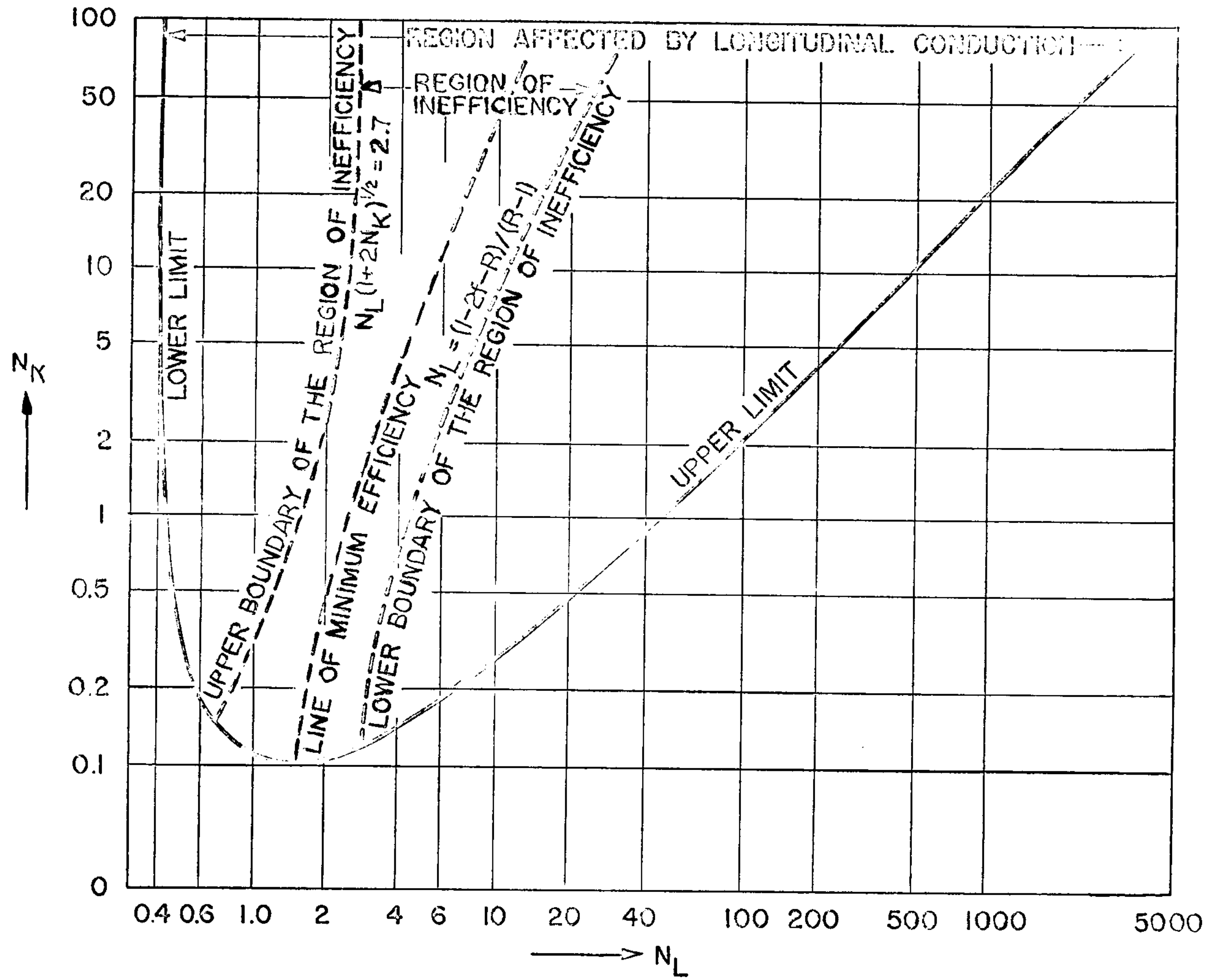

FIGURE 7 THE REGION AFFECTED BY LONGITUDINAL CONDUCTION AND THE INEFFICIENCY REGION

is plotted with respect to N_L with N_{Mo} as a parameter, References 6 and 4, a maximum is reached. The effectiveness is derived from equation (44) as follows:

$$\xi = \frac{N_L + N_L^2/[N_{Mo}(1 + N_{Mo}/N_L^2)^{1/2}]}{N_L + N_L^2/[N_{Mo}(1 + N_{Mo}/N_L^2)^{1/2}] + N_L^2/N_{Mo} + 1} \tag{73}$$

for $(N_L^2 + N_{Mo})^{1/2} > 2.7$

The expressions for maximum effectiveness and the points of inflection are lengthy.

The numerical results of the author's solution are compared with previous methods in Figure 8.

RESULTS AND CONCLUSIONS

1. The roots of the characteristic equation were graphically located and classified into two kinds - the two special roots and the three sets of one-one-one corresponding data.

2. The asymptotic relations and the effect on longitudinal conduction were related to the slope of $Y = KA_c\lambda$

$$KA_c \to 0, \qquad \beta_1 \ \& \ \beta_n \to \pm \infty$$

$$KA_c \to \infty, \qquad |\lambda_n| \to |\alpha_n|$$

3. All the roots were found to be real and distinct, except for the central curve which passes through the origin in the balanced-flow case.

4. The set of diagonal coefficients, a_{kk}, were found to be significant when longitudinal conduction was appreciable. The diagonal coefficients mathematically produce the sudden change of fluid temperature at the inlets.

The flowrate is found to have more effect than other factors.

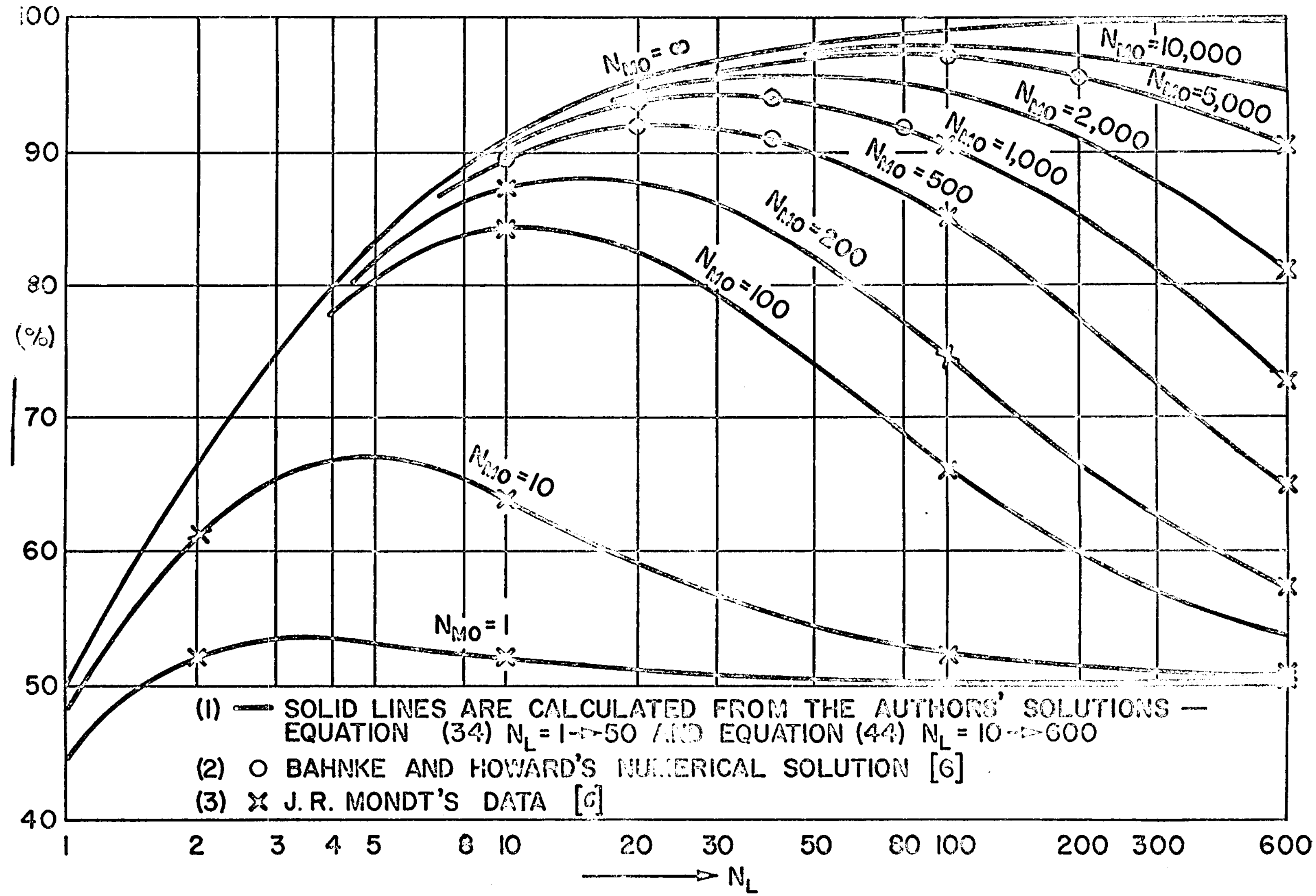

FIGURE 8 COMPARISON OF DATA WITH REFERENCE PAPERS FOR TWO-STREAM SYMMETRIC EXCHANGERS

5. A new descriptive parameter $KA_chA/(mc)^2$ was surmised mathematically.

6. A critical value $N_L = 2.7$ was determined.

7. A simplified method for the general case was determined from the order of magnitude analysis.

For the two-stream symmetric case:

8. Closed-form algebraic expressions were determined for the effectiveness and the mean and end temperature differences.

9. Length correction formulas were devised.

10. An inefficiency region was determined mathematically.

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April 27, 1966 APPROVAL TM X-53454

A NEW CONCEPT TO THE GENERAL UNDERSTANDING OF THE EFFECT OF LONGITUDINAL CONDUCTION FOR MULTISTREAM COUNTERFLOW HEAT EXCHANGERS

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This document has also been reviewed and approved for technical accuracy.

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